JOHN CURTIN
COLLEGE OF THE ARTS

YEAR 11 MATHEMATICS SPECIALIST

TEST 5, 2018
(Matrices)

Section One: Calculator Free

Student's Name: Solutions

Total Marks: 15
Time Allowed: 15 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST
Standard Items: Pens, pencils, eraser, ruler

Special Items: WACE Formula Sheet

INSTRUCTIONS TO STUDENTS
Do not open this paper until instructed to do so.
You are required to answer ALL questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

Question 1. **[4, 1, 1 = 6 marks]**

Let $\mathbf{A} = \begin{bmatrix} m & -3 \\ 4 & 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$

(a) Evaluate each of the following where possible. If not possible, state this clearly.

(i) $2\mathbf{A} - \mathrm{D}$

$$\begin{bmatrix} 2m+3 & -6 \\ 8 & 13 \end{bmatrix} \checkmark$$

(ii) **CD**

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix} \checkmark$$

(iii) **BD**

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$$

$2\times\underbrace{1 \quad 2}_{\times}\times 2$

Not possible ✓

(iv) $|\mathbf{D}|$

$-3\times 1 - 0\times 0$

$= -3$ ✓

(b) What value(s) of m make the matrix $\mathbf{A}$ singular?

$7m - -12 = 0$

$7m = -12$

$m = \frac{-12}{7}$ ✓

(c) If matrix $\mathbf{E}$ is $\begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$ and $\mathbf{AC} = \mathbf{E}$, then determine the value of m.

$$\begin{bmatrix} m & -3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$$

$m = 2$ ✓

6

Q2. **[5 marks]**

Find Matrix **A** if **AB** +2**B**=4**I**, where **I** is the identity matrix and **B**=$\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$.

$(A+2I)B = 4I$ ✓

$|B| = 6-2 = 4$ ✓

$A+2I = 4B^{-1}$

$A = 4B^{-1} - 2I$ ✓

$= 4\frac{1}{4}\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ ✓

$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ ✓

or

$AB = 4I - 2B$

$A = (4I-2B)B^{-1}$

$= 4B^{-1} - 2I$

Question 3 [1, 1, 1, 1 = 4 marks]

Let A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Find A^2, A^3, and A^4. Hence write down a formula for A^n.

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ ✓

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ ✓

$A^4 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ ✓

$\therefore A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ ✓

9

JOHN CURTIN
COLLEGE OF THE ARTS

YEAR 11 MATHEMATICS SPECIALIST

TEST 5, 2018
(Matrices)

Section Two: Calculator Assumed

Student's Name: Solutions

Total Marks: 25
Time Allowed: 25 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: Up to three approved calculators
One page (unfolded A4 sheet) front and back of Notes
WACE Formula Sheet

INSTRUCTIONS TO STUDENTS

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You are required to answer ALL questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

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Question 1 [1, 1, 2 = 4 marks]
Consider the system of equations
$2x - 3y = 3$
$4x + y = 5$
(a) Write this system of equations in matrix form, as **AX** = K

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \checkmark$$

(b) Re-write the above matrix equation in the form **X** = ...

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \checkmark$$

(c) Hence solve the system of equations using your CAS.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{9}{7} \\ -\frac{1}{7} \end{bmatrix}$$

$$\therefore \quad x = \frac{9}{7} \; \checkmark, \quad y = -\frac{1}{7} \; \checkmark$$

4

Question 2 **[2, 2, 2, 2, 2, 2, 3 = 15 marks]**

Let $A=\begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$

a) Describe the transformations represented matrix A

Dilation ∥ to x-axis s.f. of 3 ✓

and dilation ∥ to y-axis s.f. of 2 ✓

b) Describe the transformations represented matrix B

Rotation 90° Clockwise

✓ ✓

c) Object $PQRS$ is transformed by matrix A to $P'Q'R'S'$ and then by matrix B to $P"Q"R"S"$. Determine the single matrix that will transform $PQRS$ directly to $P"Q"R"S"$.

$$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}=\begin{bmatrix}0 & 2\\-3 & 0\end{bmatrix}$$

✓ ✓

d) The **point** P' is $(4\ ,-3)$ determine the coordinates of P.

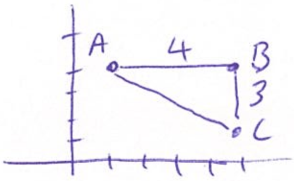
$$\begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}^{-1}\begin{bmatrix}4\\-3\end{bmatrix}=\begin{bmatrix}8/6\\-9/6\end{bmatrix}$$

✓

$P\left(\frac{4}{3}, -\frac{3}{2}\right)$ ✓

8

e) Triangle ABC with coordinates A(1,4), B(5,4) and C(5,1) is transformed by matrix A. Determine the area of the image of triangle ABC after the transformation.



$$\text{Area } \Delta = \tfrac{1}{2}(4)(3) = 6 \text{ units}^2$$

$$|A| = 6$$

$$\therefore \text{ area of } A'B'C' = 36 \text{ units}^2$$

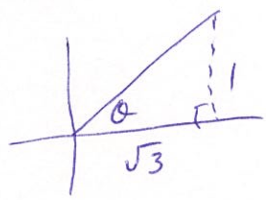
f) Matrix C represents a rotation of $\frac{\pi}{6}$ clockwise about the origin. Determine Matrix C.

$$C = \begin{bmatrix} \cos\left(-\frac{\pi}{6}\right) & -\sin\left(-\frac{\pi}{6}\right) \\ \sin\left(-\frac{\pi}{6}\right) & \cos\left(-\frac{\pi}{6}\right) \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

g) Matrix D represents a reflection in the line $y = \frac{x}{\sqrt{3}}$. Determine Matrix D.

$$m = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$



$$\begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ \sin 60^\circ & -\cos 60^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

7

Question 3 [4, 2 = 6 marks]

Suppose that A and B are 2 x 2 matrices with A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and B = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$.

(a) Prove that det (AB) = det (A) det (B)

$$AB = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$|AB| = (ae+bg)(cf+dh) - (af+bh)(ce+dg)$$
$$= a\cancel{e}cf + aedh + bgcf - a\cancel{fc}e - afdg - bhce - \cancel{bh}dg + b\cancel{g}dh$$
$$= aedh + bgcf - afdg - bhce$$

$$|A| = ad - bc \qquad |B| = eh - fg$$

$$|A||B| = (ad-bc)(eh-fg)$$
$$= adeh - adfg - bceh + bcfg$$
$$= aedh + bgfc - afdg - bhce$$
$$= |AB|$$

(b) Hence prove that if both A and B are invertible, then AB is invertible.

A&B invertible $\Rightarrow |A| \neq 0$ and $|B| \neq 0$

then $|A||B| \neq 0$

$\therefore |AB| \neq 0$ from (a)

hence AB is invertible.

6